Solution to Hands-on Problems

Example Stochastic Processes (Slide7)

- Number of commands received by a time-sharing system during time interval (0 , t1)
  - $\{X(t), 0 < t < t1\}$
  - Continuous parameter, Discrete state
- Number of students attending the *n*th lecture
  - $\{ X_n, n = 1, 2, 3, 4, \ldots\}$
  - Discrete parameter, Discrete state
- Average time to run a batch job at a computing center on the *n*th day of the week.
  - $\{ X_n, n = 1, 2, 3, 4, \ldots, 7\}$
  - Discrete parameter, Continuous state
- The waiting time of an inquiry message that arrives at time t, until processing is begun
  - $\{ X(t), t >= 0\}$
  - Continuous parameter, Continuous state

$o(h)$ notation (Slide 17)

– Application of $o(h)$

Suppose X is an Exp. r.v. with parameter λ, its c.d.f.:

$$F_X(h) = P\{X \le h\} = 1 - e^{-\lambda h}$$

What is the prob. that X is less than t+h given that it is greater than t?

$$P\{X \le t+h \mid X > t\} = P\{X \le h\}$$

$$= 1 - e^{-\lambda h} = 1 - [1 - \lambda h + \sum_{n=2}^{\infty} \frac{(-\lambda h)^n}{n!}]$$

$$= \lambda h - (\lambda h)^2 \sum_{n=2}^{\infty} \frac{(-\lambda h)^{n-2}}{n!} = \lambda h + o(h)$$

Hands-on Problems (Slides 23, 24)

- What is the mean time between failures?

  $P[\ \tau_n \leq t\ ] = 1 - e^{-\lambda t}$ where, $\lambda = 0.6$ / day

  $E[\ \tau_n\ ] = 1 / \lambda = 1.666 \ldots$ days $= 39.99 \ldots$ Hours

- The number of failures in a "t"-day interval has the Poisson distribution with a mean of $\lambda t=0.6t$. What is the probability of exactly one failure in a 24-hour period?

  $P_k(t) = P[Y_t = k] = e^{-\lambda t} (\lambda t)^k / k!$

  For t = 24 hours = 1day, k = 1 failure

$$P[Y_t = 1] = e^{-\lambda 1}\frac{(\lambda 1)^1}{1!} = e^{-0.6}(0.6)$$
$$= 0.5488 * 0.6 = 0.3293$$

- What is the probability of less than 5 failures in a week?

$$P[Y_7 < 5] = \sum_{k=0}^{4} P[Y_7 = k] = \sum_{k=0}^{4} e^{-\lambda * 7}\frac{(\lambda * 7)^k}{k!}$$
$$= \sum_{k=0}^{4} e^{-0.6*7}\frac{(0.6*7)^k}{k!} = 0.5898$$

Hands-on Problems (Slides 23, 24)

- Starting from a random point in time, what is the probability that no failure will occur during the next 24 hours?

$$P[Y_1 = 0] = P[\tau_n > 1day] = e^{-\lambda t}\frac{(\lambda t)^0}{0!}$$

$$= e^{-0.6*1} = 0.5488$$

- Suppose exactly 24 hours has elapsed with no failures. What is the expected time until the next failure?

The time between failure is exponentially distributed, by memory-less property → 1.66..days or 39.99…hours

- Four out of every five failures is a terminal problem, with equal probability on each failure. What is the process describing the terminal failure?

$$P_{terminal}\lambda = (4/5)*0.6 = 0.48 \text{ per day}$$

All component failures : $\lambda = 0.6$

Terminal failures : $\lambda * 4/5 = 0.48$

Other failures : $\lambda * 1/5 = 0.12$

Hands-on Problems (Slides 23, 24)

- What is the mean time between terminal failures?

$$1/\lambda_{terminal} = 1/0.48 = 2.0833....days$$

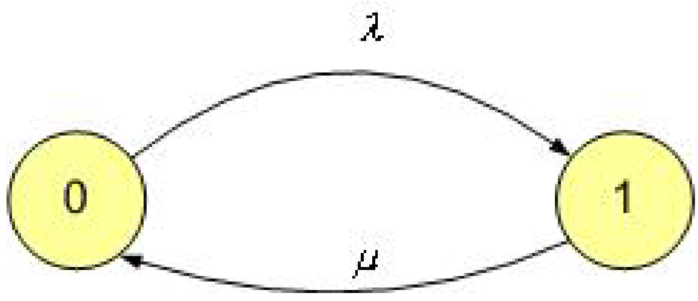
- Thus, the number of terminal failures in t days has the Poisson distribution with a mean of *0.48t*. What is the probability of *k* terminal failures in a t-day interval?

$$P[Y_t = k] = e^{-\lambda t}\frac{(\lambda t)^k}{k!} = e^{-0.48*t}\frac{(0.48*t)^k}{k!}$$

Hands-on Problems (Slide 31)

- Consider a queuing system with one server and no waiting line. And assume
  - Poisson arrivals with rate λ
  - Exponential service with rate μ

  The state transition diagram is:



Find P_0, P_1?

Balance equations: $\lambda P_0 = \mu P_1$

and

$P_0 + P_1 = 1$

So,

$$P_0 = \mu / \lambda + \mu \qquad P_1 = \lambda / \lambda + \mu$$