ECE560 Computer Systems Performance Evaluation
(Spring 2024)

Solution to Homework#2
(70 points)

Problem #1. (20 points)

Problem 23 of chap 2:

Define events:
- A: go to the Jungle Cruise
- B: ride the Monorail
- C: take the Matterhorn ride

Then: $P(A) = 0.72$ $P(B) = 0.56$ $P(C) = 0.6$

$P(A \cap B) = 0.5$ $P(A \cap C) = 0.45$ $P(B \cap C) = 0.4$ $P(A \cap B \cap C) = 0.3$

(a) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= 0.83$

(b) $P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0.5}{0.72} = 0.6944$

(c) $P(C|A \cap B) = \frac{P(A \cap B \cap C)}{P(A \cap B)} = \frac{0.3}{0.5} = 0.6$

Problem #2. (20 points)

Define Events

C: a car is a compact

U: US car

E: European Car

J: Japanese Car

$P(U) = 0.5 \quad P(E) = 0.3 \quad P(J) = 0.2$

$P(C|U) = 0.15 \quad P(C|E) = 0.4 \quad P(C|J) = 0.6$

a) $P(C) = P(C|U)P(U) + P(C|E)P(E) + P(C|J)P(J)$

$= 0.15 \times 0.5 + 0.4 \times 0.3 + 0.6 \times 0.2$

$= 0.315$

b) $P(E|C) = \dfrac{P(C|E) \cdot P(E)}{P(C)} = \dfrac{0.4 \times 0.3}{0.315} = 0.380952$

Problem #3. (30 points)

a.

Sum	First die: 1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
Second die: 3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(A) = P(\text{1st die results in a } 1/2/3) = \frac{1}{2}$

$P(B) = P(\text{2nd die results in a } 4/5/6) = 1/2$

$P(C) = P(\text{the sum of the two faces is } 7) = 6/36 = \frac{1}{6}$

$A \cap B = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\} \Rightarrow P(A \cap B) = \frac{9}{36} = \frac{1}{4}$

$A \cap C = B \cap C = A \cap B \cap C = \{(1,6), (2,5), (3,4)\} \Rightarrow$

$P(A \cap C) = P(B \cap C) = P(A \cap B \cap C) = \frac{3}{36} = \frac{1}{12}$

$\therefore P(A \cap B) = P(A) \cdot P(B) = \frac{1}{4}$

$P(A \cap C) = P(A) \cdot P(C) = \frac{1}{12}$

$P(B \cap C) = P(B) \cdot P(C) = \frac{1}{12}$

but $P(A \cap B \cap C) = \frac{1}{12} \neq P(A) \cdot P(B) \cdot P(C) = \frac{1}{24}$

Therefore, events A, B, and C are pair-wise independent, but they are not mutually independent!