ECE560: Computer Systems Performance Evaluation

Homework #3 Solution (Spring 2024)

Problem 1.

\# of arrivals per unit of time is poisson distributed with rate λ

$\Updownarrow$

inter-arrival time T is exponentially distributed with parameter λ

(a) $E[T] = \frac{1}{\lambda} = \frac{1}{13}$ minutes $= \frac{60}{13}$ seconds.

(b) $P_r\{T \le 8\} = 1 - e^{-\lambda t} = 1 - e^{-\frac{13}{60} * 8} = 0.8233$

(c) $P_r\{T > 11\} = 1 - P_r\{T \le 11\} = e^{-\lambda * 11} = e^{-\frac{13*11}{60}} = 0.0922$

(d) $P_r\{8 < T \le 11\} = F(11) - F(8) = \left(1 - e^{-\frac{13*11}{60}}\right) - \left(1 - e^{-\frac{8*13}{60}}\right)$

$= 0.1767 - 0.0922 = 0.0845$

Problem 2: Problem 41 (a)-(e) (Page97) in Chapter 2 of the Textbook by Allen

(a) Marginal p.m.f.s

$\because$ $p_X(x) = \sum_y p(x,y)$ $\quad p_Y(y) = \sum_x p(x,y)$

$\therefore$ $p_X(0) = p(0,0) + p(0,1) + p(0,2) = \frac{1}{8} + \frac{1}{4} + \frac{1}{8} = \frac{1}{2}$

$p_X(1) = p(1,0) + p(1,1) + p(1,2) = 0 + \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$p_Y(0) = p(0,0) + p(1,0) = \frac{1}{8} + 0 = \frac{1}{8}$

$p_Y(1) = p(0,1) + p(1,1) = \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$

$p_Y(2) = p(0,2) + p(1,2) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

(b) Conditional p.m.f. $p_{X|Y=2}(x,y) = \frac{p(x,2)}{p_Y(2)}$

$p_{X|Y}(0,2) = \frac{p(0,2)}{p_Y(2)} = \frac{\frac{1}{8}}{\frac{1}{2}} = \frac{1}{4}$

$p_{X|Y}(1,2) = \frac{p(1,2)}{p_Y(2)} = \frac{\frac{3}{8}}{\frac{1}{2}} = \frac{3}{4}$

(c) $p(0,0) = \frac{1}{8}$, $\quad p_X(0) = \frac{1}{2}$ $\quad p_Y(0) = \frac{1}{8}$ $\Rightarrow p_X(0) \cdot p_Y(0) = \frac{1}{16}$

$\therefore$ For at least one x and one y. $p(x,y) \ne p_X(x) \cdot p_Y(y)$

So, X and Y are not independent r.v.s.

(d) $E[X] = \sum_x x \cdot p_X(x) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}$

$E[Y] = \sum_y y \cdot p_Y(y) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{1}{2} = \frac{11}{8}$

$E[X^2] = \sum_x x^2 \cdot p_X(x) = 0^2 \cdot \frac{1}{2} + 1^2 \cdot \frac{1}{2} = \frac{1}{2}$

$E[Y^2] = \sum_y y^2 \cdot p_Y(y) = 0^2 \cdot \frac{1}{8} + 1^2 \cdot \frac{3}{8} + 2^2 \cdot \frac{1}{2} = \frac{19}{8}$

$\therefore \; Var[X] = E[X^2] - E^2[X] = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$Var[Y] = E[Y^2] - E^2[Y] = \frac{19}{8} - \left(\frac{11}{8}\right)^2 = \frac{31}{64}$

(e) p.m.f for $Z = X + Y$

$p_Z(0) = p(0,0) = \frac{1}{8}$

$p_Z(1) = p(0,1) + p(1,0) = \frac{1}{4} + 0 = \frac{1}{4}$

$p_Z(2) = p(0,2) + p(1,1) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$

$p_Z(3) = p(1,2) = \frac{3}{8}$

Problem 3.

Define r.v.s :

Y : T-T-F for the entire CS

Y_P : T-T-F for the processor subsystem

Y_m : T-T-F for the memory

Y_B : T-T-F for the Bus

X_1 : T-T-F for the P1

X_2 : T-T-F for the P2

We have :

$Y_P = \max\{X_1, X_2\}$

$Y = \min\{Y_P, Y_m, Y_B\}$

a)

$F_{Y_P} = F_{X_1} \cdot F_{X_2} = (1 - e^{-\lambda_p \cdot t})^2$

$= (1 - e^{-\frac{t}{8000}})^2$

b)

$$\Pr\{Y_p > 10000\,hrs\} = 1 - F_{Yp}(10000)$$
$$= 1 - \left(1 - e^{-\frac{10000}{8000}}\right)^2$$
$$= 1 - (1 - 0.2865)^2$$
$$= 1 - 0.5091 = \underline{\underline{0.4909}}$$

c)

According to the Min Theorem 2.7.4

(10)

$$F_Y = 1 - [1 - F_{Yp}(t)][1 - F_{Ym}(t)][1 - F_{YB}(t)]$$
$$= 1 - \left[1 - (1 - e^{-\lambda_p t})^2\right]\left[1 - (1 - e^{-\lambda_m t})\right]\left[1 - (1 - e^{-\lambda_B t})\right]$$
$$\lambda_p = \frac{1}{8000}/hr \quad \lambda_m = \frac{1}{8000}/hr \quad \lambda_B = \frac{1}{10000}/hr$$

$$F_Y(t = 10000) = 1 - 0.4909 * e^{-\frac{10000}{8000}} * e^{-\frac{10000}{10000}}$$
$$= 1 - 0.4909 * 0.2865 * 0.3679$$
$$= 0.9483$$

$$\therefore \Pr\{Y > 10000\} = 1 - F_Y(10000)$$
$$= 1 - 0.9483 = \underline{\underline{0.0517}}$$