Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth

ECE560: Computer Systems Performance Evaluation

Spring 2024

Homework #4

Name: ______________________________

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ECE560: Computer Systems Performance Evaluation (Spring 2024)
Homework #4

Assigned: February 26, Monday
Due: **March 4, Monday**

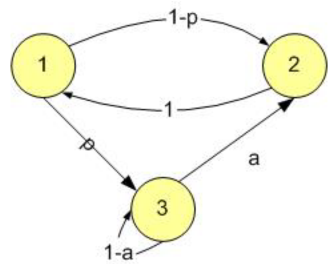
Instructions:

1. Print your name on the cover page if you choose to use it or on the first page of your answer sheets.
2. Show all steps of your solution. Answers without justification would subject to a big penalty.
3. If you submit via email, please organize all pages of your answers into one file, name your file using **"HW4-your last name.pdf or doc"** (e.g., HW4-Xing.pdf), and submit it to lxing@umassd.edu
4. Relevant lecture notes : **Lectures #9, #10, #11**

Problems:

1. A computer center has a large number of separate system components that may fail (terminals, disks, printers, sensors, etc.), without bringing the entire system down. There are, on the average, 0.01 failures per hour. Failures can be represented by a Poisson process with rate $\lambda = 0.01$/hr. What is the mean time between failures?
    b. The number of failures in a "*t*"-day interval has the Poisson distribution with a mean of $\lambda t=0.01t$. What is the probability of exactly one failure in a 1-day period?
    c. What is the probability of less than 3 failures in a week?
    d. Starting from a random point in time, what is the probability that no failure will occur during the next 1-day period?
    e. Suppose exactly 80 hours has elapsed with no failures. What is the expected time until the next failure?
    f. Three out of every five failures are a terminal problem, with equal probability on each failure.
        i. What is the process describing the terminal failure?
        ii. What is the mean time between terminal failures?
        iii. What is the probability of one terminal failure in a one-day interval?

2. Consider the homogeneous Markov chain with the following state transition diagram.



    a. Find the transition probability matrix P.
    b. Solve for the equilibrium (steady-state) probability vector Π, that is, find Π_1, Π_2, Π_3.

3. Four customers per minute enter the Frugal Fast Food restaurant during lunch hour and spend an average of 4 minutes getting their food (in a queue and receiving service), and then leave. Assume an M/M/1 model can be used. How many customers are inside the restaurant during the lunch hour, on the average?

4. **Problem 10 (a) - (f)** in Chapter 5 of the Textbook by Allen (P 344) (a copy of the problem is included below)

[12] In Hopeless Junction a small full service gas station is operated by the owner, Mirthless Snerd, by herself. On Monday mornings customers (cars) arrive randomly at the average rate of 15 per hour. Mirthless provides exponential service with a mean service time of 2.5 minutes. Please answer the following questions.

(a) What is the mean number of customers waiting (queueing) for service?

(b) What is the mean queueing time in minutes?

(c) What is the mean time a customer spends at the station?

(d) What is the mean number of customers at the station?

(e) What is the probability that Mirthless is idle?

(f) What fraction of time does Ms. Snerd have customers waiting?