Solution to Hands-on Problems in Lecture #10

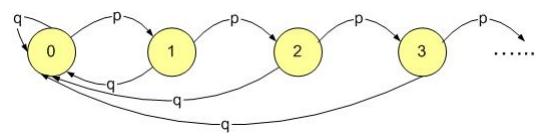
Discrete-Time MC: Example I (Slide 12)

- Consider a sequence of Bernoulli trials, for each trial
  - Success probability is p
  - Failure probability is q = 1 – p

  Assume X_n -- the state of the process at trial n, is the number of uninterrupted successes that have been completed at this point, i.e., the length of consecutive successes. Find the state transition probability matrix and state transition diagram.
- Sample:

Trial :	F	S	S	F	F	S	S	S	F
n =	0	1	2	3	4	5	6	7	8
X_n =	0	1	2	0	0	1	2	3	0

State transition diagram:

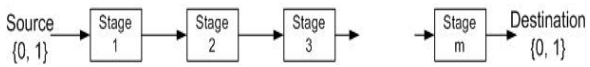


Transition probability matrix:

$$P = [P_{ij}] = \begin{bmatrix} q & p & 0 & 0 & \dots \\ q & 0 & p & 0 & \dots \\ q & 0 & 0 & p & \dots \\ q & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots \end{bmatrix}$$

Discrete-Time MC: Example II (Slide 16)

A communication system transmits the digit 0 and 1 through several stages. At each stage, there is a probability of 0.75 that the output will be the same digit as the input.



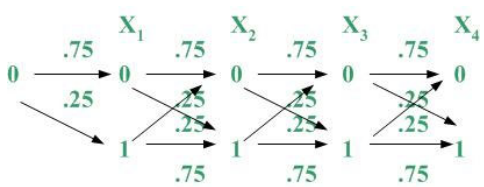
<u>What is the probability that a 0 that is entered at the first stage is output as a 0 from the 4th stage?</u>

Solution:

Let the "state" at "steps" n, X_n, denote the value output by the *n*th stage.

State Space: {0, 1}

Assume a 0 is input to stage 1



The question is now to find $P[X_4=0] = \Pi_0(4)$.

Discrete-Time MC: Example II (Cont'd)

Solution (Cont'd)

$\Pi(0) = (\Pi_0(0), \Pi_1(0))=(1, 0)$

$\Pi(1) = \Pi(0)\, P$

$\Pi(2) = \Pi(1)\, P = \Pi(0)\, P^2$

$\Pi(3) = \Pi(2)\, P = \Pi(0)\, P^3$

$\Pi(4) = \Pi(3)\, P = \Pi(0) P^4$

Given:

$$P = \begin{bmatrix} 0.75 & 0.25 \\ 0.25 & 0.75 \end{bmatrix}$$

Hence,

$$P^2 = \begin{bmatrix} 0.625 & 0.375 \\ 0.375 & 0.625 \end{bmatrix}$$

$$P^4 = \begin{bmatrix} 0.53125 & 0.46875 \\ 0.46875 & 0.53125 \end{bmatrix}$$

$\Pi(4) = (\Pi_0(4), \Pi_1(4)) = \Pi(0)P^4$

$= (1, 0)\, P^4 = (0.53125,\ 0.46875)$

So,

$\Pi_0(4) = 0.53125$