Discrete-Time MC: Example II (Slide 27)
A communication system transmits the digit 0 and 1 through several stages. At each stage, there is a probability of 0.75 that the output will be the same digit as the input.


What is the limiting probability that a 0 entered into the first stage is output as a 0 from the $\mathrm{n}^{\text {th }}$ stage as $\mathrm{n} \rightarrow \infty$ ?

## Solution:

State transition diagram Transition probability matrix


$$
P=\left[\begin{array}{ll}
0.75 & 0.25 \\
0.25 & 0.75
\end{array}\right]
$$

Clearly, this MC is aperiodic and irreducible, and thus ergodic!

$$
\begin{aligned}
& \text { So, it has a steady state distribution: } \\
& \begin{array}{l}
\text { So, it has a steady state distribution: } \\
\Pi=\Pi^{*} \mathrm{P} \rightarrow\left(\Pi_{0}, \Pi_{1}\right)=\left(\Pi_{0}, \Pi_{1}\right) *\left[\begin{array}{ll}
0.75 & 0.25 \\
0.25 & 0.75
\end{array}\right] \rightarrow
\end{array} \\
& \left.\begin{array}{l}
\Pi_{0}=0.75 \Pi_{0}+0.25 \Pi_{1} \\
\Pi_{1}=0.25 \Pi_{0}+0.75 \Pi_{1} \quad \text { and } \\
\Pi_{0}+\Pi_{1}=1
\end{array}\right\} \rightarrow \begin{array}{l}
\Pi_{0}=\Pi_{1}=0.5 \\
\Pi=(0.5,0.5)
\end{array}
\end{aligned}
$$

Thus:

$$
\lim _{n \rightarrow \infty} \Pi_{0}(n)=\Pi_{0}=0.5
$$

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Markov Processes

## Exercise (Slide 29)

Write down the balance equations for the following MC:

$P_{11}$

Rate leaving $=$ rate entering
State
0 :
$\Pi_{0}\left(\mathrm{P}_{00}+\mathrm{P}_{01}+\mathrm{P}_{02}\right)=\Pi_{0} \mathrm{P}_{00}+\Pi_{1} \mathrm{P}_{10}+\Pi_{2} \mathrm{P}_{20}$

1 :
$\Pi_{1}\left(\mathrm{P}_{10}+\mathrm{P}_{11}+\mathrm{P}_{12}\right)=\Pi_{0} \mathrm{P}_{01}+\Pi_{1} \mathrm{P}_{11}+\Pi_{2} \mathrm{P}_{21}$
2 :
$\Pi_{2}\left(\mathrm{P}_{20}+\mathrm{P}_{21}+\mathrm{P}_{22}\right)=\Pi_{0} \mathrm{P}_{02}+\Pi_{1} \mathrm{P}_{12}+\Pi_{2} \mathrm{P}_{22}$
$\vartheta$
$\left(\begin{array}{lll}\Pi_{0} & \Pi_{1} & \Pi_{2}\end{array}\right)=\left(\begin{array}{llll}\Pi_{0} & \Pi_{1} \Pi_{2}\end{array}\right)\left(\begin{array}{llll}\mathrm{P}_{00} & \mathrm{P}_{01} & \mathrm{P}_{02} \\ \mathrm{P}_{10} & \mathrm{P}_{11} & \mathrm{P}_{12} \\ \mathrm{P}_{20} & \mathrm{P}_{21} & \mathrm{P}_{22}\end{array}\right]$

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