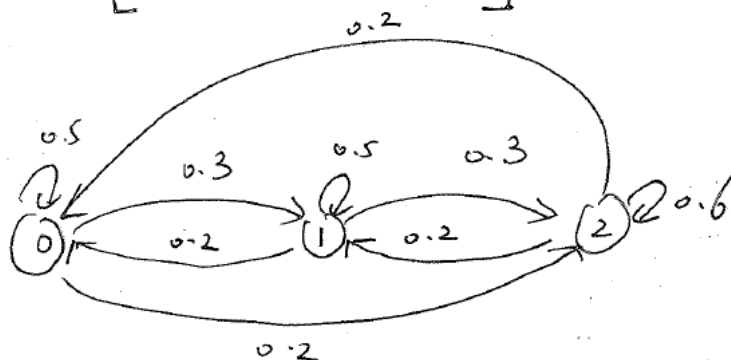


Solution to Problems on Slide 30

a)



$$P = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix}$$



b)

ergodic, because MC is irreducible,
aperiodic, and $\left\{ \begin{array}{l} \text{has finite state space, or} \\ \text{positive recurrent} \end{array} \right.$

c)

$$\begin{aligned} \pi(1) &= \pi(0) \cdot P \\ &= [0.2 \ 0.5 \ 0.3] \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \\ &= [\underline{0.26} \ 0.37 \ 0.37] \end{aligned}$$

so, the prob (Great year after 1 year) is 0.26.

d)

$$\begin{cases} \pi = \pi P \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} [\pi_0 \ \pi_1 \ \pi_2] = [\pi_0 \ \pi_1 \ \pi_2] \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.6 \end{bmatrix} \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$$\Rightarrow \pi_0 = 0.28571 \quad \pi_1 = 0.32653 \quad \pi_2 = 0.38776$$

So, prob (Greet you after n year as $n \rightarrow \infty$) is

$$0.28571$$