| Solution to Problem on Slide |
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| 36 |
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## M/M/1/k/k Queues (VI): Example

- Users of a time-sharing system have exponentially distributed think times with an average value of 20 seconds.
- Service time are exponentially distributed with a mean of 1.5 seconds.
- Q: how many terminals can the system support if we want the average stretch factor is no more than 5?
- Solution: Use $\mathrm{M} / \mathrm{M} / 1 / \mathrm{k} / \mathrm{k}$ model

$\beta=$ arrival rate per user $=1 / 20$ per second
$\mu=$ service rate $=1 / 1.5=2 / 3$ per second
Find maximum k satisfying $\mathrm{W} / \mathrm{Ws}<=5$ ?

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M/M/1/k/k Queues (VI): Example

- Solution (Cont'd):

$$
\begin{aligned}
& \left.\begin{array}{l}
W=k / \lambda-1 / \beta \\
W_{s}=1 / \mu \\
\lambda=\rho \mu
\end{array}\right\} \Rightarrow \frac{W}{W_{s}}=\frac{k / \lambda-1 / \beta}{1 / \mu}=\frac{k}{\rho}-\frac{\mu}{\beta} \leq 5 \\
& \therefore \frac{k}{\rho} \leq 5+\frac{2 / 3}{1 / 20}=18.33
\end{aligned}
$$

Where, $\rho=1-B[k, z]$ and $z=\mu / \beta=\frac{2 / 3}{1 / 20}=13.33$
Using the figures below:

| $k$ | $B[k, 13.33]$ | $\rho$ | $k / \rho$ |
| :---: | :---: | :---: | :---: |
| 18 | 0.048 | 0.952 | 18.9 |
| 17 | 0.07 | 0.93 | 18.23 |

. $k$ should not exceed 17 !

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