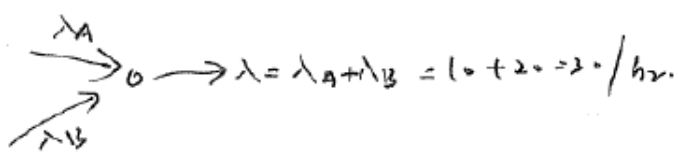
ECE560 Midterm Sample Questions Solution
Spring 2024

1. **Hands-on Problem in L#3, Slides#23, 34, 35**
   Homework #1 problem
   Review question in L#4, Slide #4
   Refer to course website Lecture and Homework sections for solutions

2. **(Extra-credit question in L#5)** A factory production line is manufacturing bolts using three machines, A, B and C. Of the total output, machine A is responsible for 20%, machine B for 30% and machine C for the remaining 50%. It is known from previous testing data that 3% of the output from machine A, 5% from machine B and 2% from machine C is defective.
   a) What is the probability that a randomly chosen bolt is defective?
   b) A bolt is chosen at random from the production line and found to be defective. What is the probability that this bolt came from machine B?
   Similar questions:
   Hands-on Problem in L#5, Slide#22
   Homework #2 Problems #1, #2
   Refer to course website Lecture and Homework sections for solutions

3. Consider a server with a job-arrival stream that is a superposition of two independent Poisson sub-streams: one for *type-A* jobs with an average rate of λ_A=10/hour, the other for *type-B* jobs with an average rate of λ_B=20/hour.
   a) Suppose exactly 10 hours have elapsed with no job arrivals. What is the expected time in minutes until the next *type-B* job arrival?
   b) What is the probability that two *type-B* jobs arrive in a 4-minute period?
   c) What is the mean time between successive job arrivals for the server? Here the job can be either *type-A* or *type-B*.
   d) What is the probability that the time interval between successive job arrivals is longer 3 minutes? Here the job can be either *type-A* or *type-B*.



$\lambda = \lambda_A + \lambda_B = 10 + 20 = 30 / hr.$

(a) Memory less property:

$$MTTF_B = \frac{1}{\lambda_B} = \frac{1}{20}\ hr = 3\ min.$$

(b)
$$Pr\{Y_B = 2\} = e^{-\lambda_B t}\frac{(\lambda_B t)^2}{2!}$$
$$= e^{-\frac{20}{60}*4}\frac{(\frac{20}{60}*4)^2}{2!}$$
$$= e^{-\frac{4}{3}}\left(\frac{4}{3}\right)^2 / 2$$
$$= 0.2343$$

(c) $\lambda = \lambda_A + \lambda_B = 30/hr$

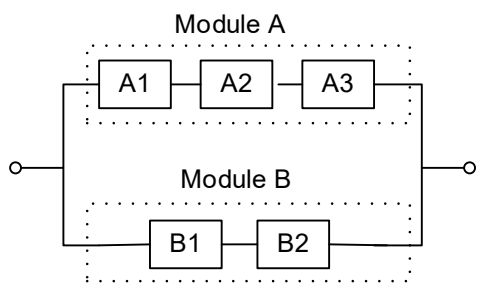
$MTTF = \frac{1}{\lambda} = \frac{1}{30}\,hr = \frac{60}{30}\,min = 2\,min$

(d) $P\{\tau > 3\} = 1 - F(3) = e^{-\lambda * 3} = e^{-\frac{30}{60} * 3}$

$= e^{-\frac{3}{2}} = 0.22313$

Similar questions:
Hands-on Problem in L#9, Slides#23-24
Homework #3 Problem #1
Homework #4 Problem #1
Refer to course website Lecture and Homework sections for solutions

4. A system consists of two modules A and B as shown in the following figure. The entire system fails when both of the two modules have failed. Module A consists of three components A1, A2, and A3, and it fails if any of A1, A2, A3 fails. Module B consists of two components B1, and B2, and it fails if any of B1, B2 fails.
   Assume all the system components (A1, A2, A3, B1, B2) have the same exponential time-to-failure with a mean of 6000 hours, and each component fails independently.



Answer the following questions:

a) What is the probability that the *Module A* will not fail during 2000 hours of continuous operation?
b) What is the probability that the *Module B* will not fail during 2000 hours of continuous operation?
c) What is the probability that the *entire system* will not fail during 2000 hours of continuous operation?

$\lambda_{A1} = \lambda_{A2} = \lambda_{A3} = \lambda_{B1} = \lambda_{B2} = \lambda = \frac{1}{6000}/hr$

$F_{A1} = F_{A2} = F_{A3} = F_{B1} = F_{B2} = 1 - e^{-\lambda t}$

a) $Y_A = \min\{A_1, A_2, A_3\}$

$F_{Y_A} = 1-(1-F_{A_1})(1-F_{A_2})(1-F_{A_3}) = 1-e^{-3\lambda t}$

$P_r\{\tau_A > 2000\} = 1 - F_{Y_A}(t=2000) = e^{-3\lambda t} = e^{-1} = 0.367879$

b) $Y_B = \min\{B_1, B_2\}$

$F_{Y_B} = 1-(1-F_{B_1})(1-F_{B_2}) = 1-e^{-2\lambda t}$

$P_r\{\tau_B > 2000\} = 1 - F_{Y_A}(t=2000) = e^{-2\lambda t} = e^{-\frac{4000}{6000}} = 0.513417$

c) $Y = \max\{Y_A, Y_B\}$ $\quad F_Y = F_{Y_A} F_{Y_B} = (1-e^{-3\lambda t})(1-e^{-2\lambda t})$

$P_r\{\tau > 2000\} = 1 - F_Y(t=2000) = 1-(1-0.367879)(1-0.513417)$
$= 0.692421$

Similar questions:
Homework #3 Problem #3
Hands-on Problems in L#7, Slides#19, #21, #22
Refer to course website Lecture and Homework sections for solutions

5. A web server can be in an *idle* state (state 0) or a *busy* state (state 1). Observing its state at 3pm each day, we believe that the web server approximately behaves like a homogeneous Markov chain with the following one-step transition probability matrix:

$$P = \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix}$$

Suppose the initial probability for being in the *idle* state is 0.3, and for being in the *busy* state is 0.7. Answer the following questions:

a) What is the probability that the web server is in the busy state after 2 days?
b) What is the probability that the web server is in the busy state after n days as $n \rightarrow \infty$?

a) $\pi(2) = (\pi_0(2), \pi_1(2)) = \pi(0) \cdot P^2 = (\pi_0(0), \pi_1(0)) P^2$

$$= (0.3 \quad 0.7) \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix}^2$$

$$P^2 = \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix} \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2 \cdot 0.2 + 0.8 \cdot 0.1 & 0.2 \cdot 0.8 + 0.8 \cdot 0.9 \\ 0.1 \cdot 0.2 + 0.9 \cdot 0.1 & 0.1 \cdot 0.8 + 0.9 \cdot 0.9 \end{bmatrix} = \begin{bmatrix} 0.12 & 0.88 \\ 0.11 & 0.89 \end{bmatrix}$$

$$\pi(2) = (\pi_0(2) \quad \pi_1(2)) = (0.3 \quad 0.7) \begin{bmatrix} 0.12 & 0.88 \\ 0.11 & 0.89 \end{bmatrix}$$

$$= [0.113 \quad 0.887]$$

$$\underline{\underline{\pi_1(2) = 0.887}}$$

b) $\begin{cases} \pi = \pi P \\ \pi_0 + \pi_1 = 1 \end{cases}$ $\quad (\pi_0 \ \pi_1) = (\pi_0 \ \pi_1) \begin{bmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{bmatrix}$

$$\begin{cases} \pi_0 = 0.2\pi_0 + 0.1\pi_1 & (1) \\ \pi_0 + \pi_1 = 1 & (2) \end{cases}$$

By (1): $0.8\pi_0 = 0.1\pi_1 \Rightarrow \pi_1 = 8\pi_0$

By (2): $\pi_0 + 8\pi_0 = 1 \Rightarrow \pi_0 = \frac{1}{9}$

$$\pi_1 = \frac{8}{9}$$

$$\underline{\underline{\pi_1 = \frac{8}{9} = 0.889}}$$

Similar questions:
Hands-on Problems in L#10, Slides#17, #27, #30
Homework #4 Problems #2
Refer to course website Lecture and Homework sections for solutions