ECE560 Extra-Credit Question

- Consider a computer system with one processor and a queue with 1 buffer. The job requests arrive to the processor at the rate of 16 requests per second with Poisson pattern. The time to service a job request at the processor is exponentially distributed with a mean of 50 milliseconds. Assume a job request in the queue and not being serviced can depart without service; this behavior is called "defect". Assume the defect process is also exponential with the constant rate of δ = 2 requests/second.
  - Draw the complete state-transition diagram.
  - What is the probability of the entire system is idle?
  - What is the effective arrival rate of job requests into the system?
  - What is the average number of job requests in the system?
  - What is the average response time for a job?
  - What is the average waiting time in the queue for a job?

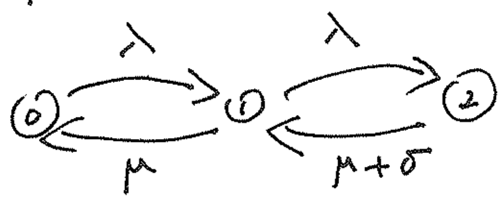
$\lambda = 16/sec$

$W_s = 50ms$

$\mu = \frac{1}{W_s} = 20/sec$

$\delta = 2/sec$

① $M/M/1/2$ $\quad N=2$



②

state	rate in = rate out	
0	$\mu \pi_1 = \lambda \pi_0$	... ⓐ
1	$\lambda \pi_0 + (\mu+\delta)\pi_2 = (\lambda+\mu)\pi_1$	... ⓑ
2	$\lambda \pi_1 = (\mu+\delta)\pi_2$	... ⓒ
		ⓓ

per ⓐ : $\pi_1 = \frac{\lambda}{\mu}\pi_0$

per ⓒ : $\pi_2 = \frac{\lambda}{\mu+\delta}\pi_1 = \frac{\lambda^2}{\mu(\mu+\delta)}\pi_0$ ... ⓔ

$$\pi_0 + \pi_1 + \pi_2 = 1$$

$$\pi_0\left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu(\mu+\delta)}\right) = 1$$

$$\pi_0 = \frac{1}{1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu(\mu+\delta)}} = \frac{1}{2.38182} = 0.4198$$

③ $\rho = 1 - \pi_0 = \dfrac{\lambda_{eff}}{\mu}$

$\underline{\underline{\lambda_{eff}}} = \mu(1-\pi_0) = 20 * (1-0.4198) = 11.6031$

④ $\pi_0 = 0.4198$

per ⓓ $\pi_1 = \dfrac{\lambda}{\mu}\pi_0 = 0.3359$

per ⓔ $\pi_2 = \dfrac{\lambda^2}{\mu(\mu+\delta)}\pi_0 = 0.5818 * \pi_0 = 0.2443$

$$L = \sum_{n=0}^{2} n\pi_n = 0\pi_0 + 1\pi_1 + 2\cdot\pi_2 = 0.3359 + 2 * 0.2443$$

$$= 0.8244$$

⑤ $W = \dfrac{L}{\lambda_{eff}} = \dfrac{0.8244}{11.6031} = 0.071$ sec.

⑥ $W_Q = W - W_s = 0.071 - 0.05 = 0.021$ sec

$= 21$ ms