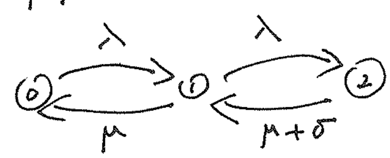
ECE560 Final Exam Sample Questions Solution (Spring 2024)
Dr. Liudong Xing

1. Lecture #13, Hands-on problems on Slides #5 & #16 (**M/M/1/N**)
   Refer to solutions on the course website

2. (**L#13 Extra Credit Question**) Consider a computer system with one processor and a queue with 1 buffer. The job requests arrive to the processor at the rate of 16 requests per second with Poisson pattern. The time to service a job request at the processor is exponentially distributed with a mean of 50 milliseconds. Assume a job request in the queue and not being serviced can depart without service; this behavior is called "defect". Assume the defect process is also exponential with the constant rate of δ = 2 requests/second.
   a) Draw the complete state-transition diagram.
   b) What is the probability of the entire system is idle?
   c) What is the effective arrival rate of job requests into the system?
   d) What is the average number of job requests in the system?
   e) What is the average response time for a job?
   f) What is the average waiting time in the queue for a job?

(1) $M/M/1/2$ $N=2$ $\delta = 2/sec$



(2)

state	rate in = rate out	
0	$\mu\pi_1 = \lambda\pi_0$	... (a)
1	$\lambda\pi_0 + (\mu+\delta)\pi_2 = (\lambda+\mu)\pi_1$	... (b)
2	$\lambda\pi_1 = (\mu+\delta)\pi_2$	... (c)

(d)

per (a): $\pi_1 = \frac{\lambda}{\mu}\pi_0$

per (c): $\pi_2 = \frac{\lambda}{\mu+\delta}\pi_1 = \frac{\lambda^2}{\mu(\mu+\delta)}\pi_0$ ... (e)

$$\pi_0 + \pi_1 + \pi_2 = 1$$

$$\pi_0\left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu(\mu+\delta)}\right) = 1$$

$$\pi_0 = \frac{1}{1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu(\mu+\delta)}} = \frac{1}{2.38182} = 0.4198$$

③ $\rho = 1 - \pi_0 = \frac{\lambda_{eff}}{\mu}$

$\lambda_{eff} = \mu(1-\pi_0) = 20 * (1-0.4198) = 11.6031$

④ $\pi_0 = 0.4198$

per (d) $\pi_1 = \frac{\lambda}{\mu}\pi_0 = 0.3359$

per (e) $\pi_2 = \frac{\lambda^2}{\mu(\mu+\delta)}\pi_0 = 0.5818 * \pi_0 = 0.2443$

$L = \sum_{n=0}^{2} n\pi_n = 0\pi_0 + 1\pi_1 + 2\cdot\pi_2 = 0.3359 + 2 * 0.2443$
$= 0.8244$

⑤ $W = \frac{L}{\lambda_{eff}} = \frac{0.8244}{11.6031} = 0.071$ sec.

⑥ $W_Q = W - W_s = 0.071 - 0.05 = 0.021$ sec
$= 21$ ms

Similar questions: HW#5, Problem#1
Refer to solutions on the course website

3. Lecture #13, Hands-on problems on Slide #24 (M/M/c)
   Similar questions: HW#5, Problem#2
   Refer to solutions on the course website

4. (**L#14 hands-on problem, Slide 24**) Considering a computer subsystem that can be modeled as the GI/M/1 queueing system. Specifically, the service time is exponentially distributed with a constant rate of 60 jobs per second. The Laplace-Stieltjes transform of the job inter-arrival time τ is assumed to be $A^*[\theta] = \dfrac{\theta + \lambda}{3\lambda}$ with $\lambda = 30$.
   a) What is the probability that an arriving job finds the system is busy?
   b) What is the probability that an arriving job finds 3 customers in the system?
   c) What is the average number of jobs in the system queue?
   d) What is the average time a job spends in the system?
   e) What is the probability that the system is busy?
   f) What is the probability that there are 3 jobs in the system?

(a) $1-\pi_0 = A^*[\mu\pi_0] = \dfrac{\mu\pi_0+\lambda}{3\lambda}$

$\mu = 60$ jobs/sec

$\rho = \dfrac{\lambda}{\mu} = \dfrac{30}{60} = 0.5$

$3\lambda - 3\lambda\pi_0 = \mu\pi_0 + \lambda$

$2\lambda = (\mu+3\lambda)\pi_0$

$\pi_0 = \dfrac{2\lambda}{\mu+3\lambda} = \dfrac{60}{60+90} = \dfrac{60}{150} = 0.4$

$1-\pi_0 = 0.6$

$\Rightarrow$ the probability that an arriving job finds the system is busy is 60%

(b) $\pi_3 = \pi_0(1-\pi_0)^3$

$= 0.4 * 0.6^3 = 0.0864$

$\Rightarrow$ The prob that an arriving job finds 3 customers in the system is 0.0864.

(c) $L_q = \dfrac{\rho(1-\pi_0)}{\pi_0} = \dfrac{0.5 * 0.6}{0.4} = 0.75$

(d) $W = \frac{\rho}{\pi_0} / \lambda = \frac{w_s}{\pi_s} = \frac{1}{\mu \cdot \pi_0} = \frac{1}{60 * 0.4} = \frac{1}{24} = 0.042$ second

(e) : prob that the system is busy (from a random observer point-of-view)

$p_0 = 1 - \rho = 1 - 0.5 = 0.5$

(Note difference between (a) (e))

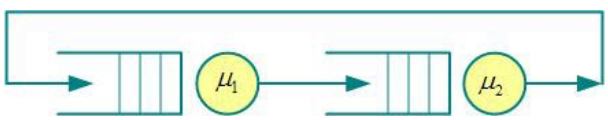
(f) prob that there are 3 jobs in the system

$p_3 = \rho \pi_0 (1-\pi_0)^2 = 0.5 * 0.4 * 0.6^2 = 0.072$ (compared to (b))

Similar questions: HW#6, Problem#1

Refer to solutions on the course website

5. Consider the following closed queuing network with 2 nodes:



Assume

- Number of customers in the system is N=3
- The service times are exponentially distributed with $\mu_1 = 3$ jobs/sec, $\mu_2 = 2$ jobs/sec
- Scheduling disciplines are FCFS for both nodes
- Transition probabilities are $p_{12} = 1$, $p_{21} = 1$

Answer the following questions:

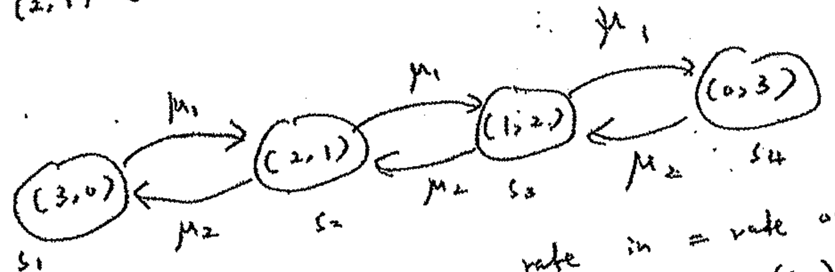
a) Draw the state-transition diagram
b) Find the local balance equation(s) for each state?
c) Find the joint steady-state probabilities for all states?
d) Use $\pi_i(n)$ to denote the marginal steady-state probability that there are *n* jobs in node *i*. Find the following marginal steady-state probabilities: $\pi_1(0)$, $\pi_1(3)$, $\pi_2(1)$, $\pi_2(4)$

$N = 3 \quad K = 2$ total # of

$C_{N+K-1}^{K-1} = C_4^1 = 4$ and state space is:

$\{(3,0) \ (2,1) \ (1,2) \ (0,3)\}$ simply denoted by $\{s_1, s_2, s_3, s_4\}$

a)



b)

state		node
s_1	(3,0)	1
s_2	(2,1)	1, 2
s_3	(1,2)	1, 2
s_4	(0,3)	2

rate in = rate out

$\pi(2,1)\mu_2 = \pi(3,0)\mu_1$ ①

$\pi(1,2)\mu_2 = \pi(2,1)\mu_1$ ②

$\pi(3,0)\cdot\mu_1 = \pi(2,1)\mu_2$

$\pi(0,3)\cdot\mu_2 = \pi(1,2)\cdot\mu_1$ ③

$\pi(2,1)\mu_1 = \pi(1,2)\cdot\mu_2$

$\pi(1,2)\mu_1 = \pi(0,3)\mu_2$

c)

By ① $\quad \pi(2,1) = \dfrac{\mu_1}{\mu_2}\cdot\pi(3,0)$

By ② $\quad \pi(1,2) = \dfrac{\mu_1}{\mu_2}\cdot\pi(2,1) = \left(\dfrac{\mu_1}{\mu_2}\right)^2\pi(3,0)$

By ③ $\quad \pi(0,3) = \dfrac{\mu_1}{\mu_2}\cdot\pi(1,2) = \left(\dfrac{\mu_1}{\mu_2}\right)^3\cdot\pi(3,0)$

$$\pi(3,0) + \pi(2,1) + \pi(1,2) + \pi(0,3) = 1$$

$$\Rightarrow \pi(3,0) = \frac{1}{1 + \frac{\mu_1}{\mu_2} + \left(\frac{\mu_1}{\mu_2}\right)^2 + \left(\frac{\mu_1}{\mu_2}\right)^3}$$

$$= \frac{1}{1 + 1.5 + 1.5^2 + 1.5^3} = \frac{1}{8.125}$$

$$= 0.1231$$

$$\Rightarrow \pi(2,1) = 1.5 * \pi(3,0) = 0.1846$$

$$\Rightarrow \pi(1,2) = 1.5 * \pi(2,1) = 0.2769$$

$$\Rightarrow \pi(0,3) = 1.5 * \pi(1,2) = 0.4154$$

(d) $\pi_1(0) = \pi(0,3) = 0.4154$

$\pi_1(3) = \pi(3,0) = 0.1231$

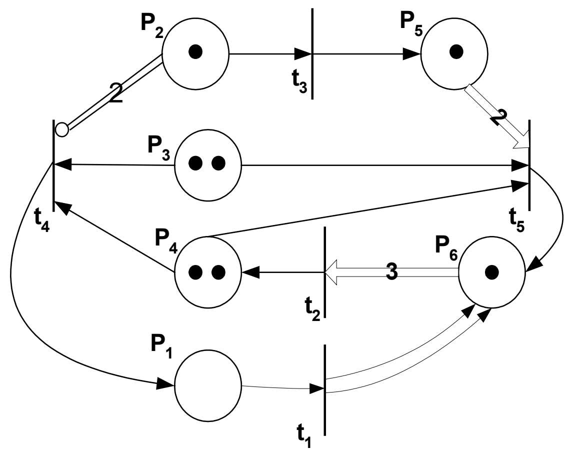
$\pi_2(1) = \pi(2,1) = 0.1846$

$\pi_2(4) = 0$ (impossible!)

Similar questions: Lecture #15 Slide #44
Refer to solutions on the course website

6. Lecture #15, Hands-on problems on Slides #12, 15, 24, 28&29
Another sample question on open QNs: HW#6, Problem#3
Refer to solutions on the course website

7. In a Petri net model, an inhibitor arc from place P to transition t modifies the enabling rule in the sense that the transition can fire only if place P does not contain tokens. Multiple (for example n) inhibitor arcs between an input place and a transition implies inhibition threshold, meaning at least n tokens are needed to inhibit transition firing; or when the place has less than n tokens, the transition can be firable. For the Petri net below,



a) Specify the input function **I** and the output function **O,** ignoring the inhibitor arc between P_2 and t_4, which is denoted by inhibitor function H: H $(t_4)=\{P_2, P_2\}$.

b) What is the current marking M of the Petri net?

c) Which transition(s) are enabled in the current marking M? What would the state of the Petri net be after firing?

d) Modify the Petri net by replacing the inhibitor arc between P_2 and t_4 with <u>a directed arc of multiplicity of 1 from P_2 to t_1</u>. The following questions refer to the Petri net after the above modification.

   1) Specify the input matrix $\mathbf{D}^-$, the output matrix $\mathbf{D}^+$, and the incidence matrix **D** for the *modified* Petri net
   2) Is this *modified* Petri net conservative? Why or why not? Explain using **Matrix analysis** method!

a) $I(t_1)=\{P_1\}$ $I(t_2)=\{P_6,P_6,P_6\}$ $I(t_3)=\{P_2\}$ $I(t_4)=\{P_3,P_4\}$

$I(t_5)=\{P_3,P_4,P_5,P_5\}$

$O(t_1)=\{P_6,P_6\}$ $O(t_2)=\{P_4\}$ $O(t_3)=\{P_5\}$ $O(t_4)=\{P_1\}$

$O(t_5)=\{P_6\}$

b) $M=(0\ 1\ 2\ 2\ 1\ 1)$

c) t_4, t_3 6 places

$$M = (0\ 1\ 2\ 2\ 1\ 1) \xrightarrow{t_3 \text{ fired}} (0\ 0\ 2\ 2\ 2\ 1)$$

$$M = (0\ 1\ 2\ 2\ 1\ 1) \xrightarrow{t_4 \text{ fired}} (1\ 1\ 1\ 1\ 1\ 1)$$

d) 1)

$$D^- = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 & 0 \end{bmatrix} \end{array}$$

$$D^+ = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$D = \begin{array}{c} t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \end{array} \begin{bmatrix} -1 & -1 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 0 & -3 \\ 0 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & -1 & -1 & -2 & 1 \end{bmatrix}$$

$$D \cdot W_p^T = D_{5\times 6}\begin{bmatrix}1\\1\\1\\1\\1\\1\end{bmatrix}_{6\times 1} = \begin{bmatrix}0\\-2\\0\\-1\\-3\end{bmatrix}_{5\times 1} \neq \begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}_{5\times 1}$$

Therefore, this modified PN is not conservative!