Department of Electrical and Computer Engineering
University of Massachusetts Dartmouth

ECE560: Computer Systems Performance Evaluation

Spring 2024

Homework #6

Name: ______________________________

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ECE560: Computer Systems Performance Evaluation (Spring 2024)
Homework #6

Assigned: April 1, Monday
Due: **April 8, Monday**

Instructions:

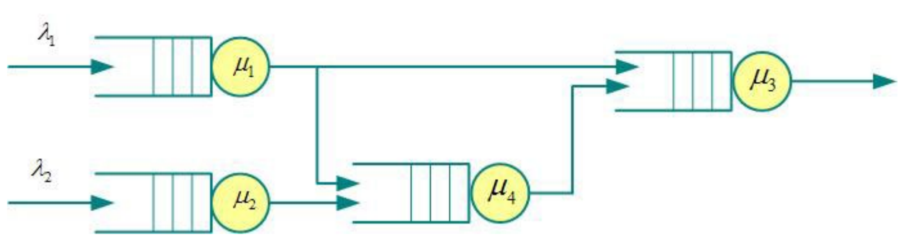
1. Print your name on the cover page if you choose to use it or on the first page of your answer sheets.
2. Show all steps of your solution. Answers without justification would subject to a big penalty.
3. Please organize all pages of your answers into one file, name your file using **"HW6-your last name.pdf or doc"** (e.g., HW6-Xing.pdf), and submit it to lxing@umassd.edu
4. Relevant lecture notes : **Lecture#14, 15**

Problems:

1. Consider the GI/M/1 queueing systems, where the inter-arrival time τ has an Erlang-2 distribution with parameter λ (refer to Chapter 3.2.6 about the Erlang-k distribution, its LST is $A^*[\theta] = \left(\frac{k\lambda}{k\lambda + \theta}\right)^k$, here in the problem k=2), the service time is exponentially distributed with a mean of $1/\mu$.
   a) What is the steady-state probability that an arriving customer finds the system empty?
   b) If λ is 15 per hour and μ is 20 per hour, what is the probability that an arriving customer finds 4 customers in the system?
   c) What is the average number of customers in the system?
   d) What is the average waiting time of a customer in the queue of the system?

2. Lili Malign, an analyst at Luigi's Contract Service, is considering the queue discipline to use for one of the main office systems. Lili modeled this system as an M/D/1 queueing system with λ=5 customers per hour and W_s=9 minutes.
   Help her calculate L, W, L_q, W_q assuming
   a) FCFS (First-come, first-served), and
   b) LCFS (Last-come, first-served) non-preemptive queue discipline.

3. **Jackson Theorem/Algorithm:** Consider the following open queueing network:



Given that

- Number of nodes K=4
- Service times are exponentially distributed with
  $\frac{1}{\mu_1} = 0.1$, $\frac{1}{\mu_2} = 0.4$, $\frac{1}{\mu_3} = 0.1$, $\frac{1}{\mu_4} = 0.15$
- The arrival to node 1 has Poisson pattern with rate $\lambda_1 = 5\, jobs/\sec$
- The inter-arrival time to node 2 is exponentially distributed with parameter $\lambda_2 = 2\, jobs/\sec$
- FCFS scheduling discipline is used for all nodes
- Transition probabilities are
  $p_{13} = 0.4$, $p_{14} = 0.6$, $p_{24} = 1$, $p_{43} = 1$

Answer the following questions:

a) Find the average arrival rates to each node?

b) Find the following performance measures:

   (1) Server utilization for each node

   (2) Mean response time for each node

   (3) Mean number of jobs at each node

   (4) Mean overall response time of the system

   (5) Marginal steady-state probabilities $\pi_1(1)$, $\pi_2(2)$, $\pi_3(3)$, $\pi_4(4)$

c) Find the joint steady-state probability $\pi(1,2,3,4)$?